Phase transitions and bubble nucleations for a ϕ^6 model in curved spacetime

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Abstract

Condsidering a massive self-interacting ϕ^6 scalar field coupled arbitrarily to a (2+1) dimensional Bianchi type-I spacetime, we evaluate the one-loop effective potential. It is found that ϕ^6 potential can be regularized in (2+1) dimensional curved spacetime. A finite expression for the energy-momentum tensor is obtained for this model. Evaluating the finite temperature effective potential, the temperature dependence of phase transitions is studied. The crucial dependence of the phase transitions on the spacetime curvature and on the coupling to gravity are also verified. We also discuss the nucleation of bubbles in a ϕ^6 model. It is found that there exists an exact solution for the damped motion of the bubble in the thin wall regime.

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Gravity in (2+1) dimensions [1] exhibits novel features of interest. There are several important differences between the three and four dimensional

problems. First of all the divergence in the gravitation action induced by scalar loops in 4 dimensions can, by power counting be proportional to 1, R, R^2, $R_{\mu\nu}$ $R^{\mu\nu}$ and $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$(or suitable combinations of them.). In three dimensions the situation is simplified, as the only candidates are 1 and R [2]. Over the past two decades (2 + 1) dimensional gravity has become an active field of research, drawing insights from general relativity. The task of quantizing general relativity remains one of the outstanding problems of theoretical physics. General relativity is a geometric theory of spacetime, and quantising gravity means quantising spacetime itself. Ordinary quantum field theory is local, but the fundamental physical observables of quantum gravity are necessarily nonlocal. Ordinary quantum field theory takes causality as a fundamental postulate, but in quantum gravity the spacetime geometry and thus the light cones and the causal structure, are themselves subject to quantum fluctuations. Again, perturbative quantum field theory depends on the existence of a smooth, approximately flat background, but there is no reason to believe that the short-distance limit of quantum gravity even resembles a smooth manifold. Faced with these problems, it is natural to look for simpler models that share the important conceptual features of general relativity while avoiding some of the conceptual difficulties. General relativ-

ity in (2 + 1) dimensions is one such model. As a generally covariant theory of spacetime geometry, (2 + 1) dimensional gravity has the same conceptual foundations as relativistic (3 + 1) dimensional general relativity, and many of the fundamental issues of quantum gravity carry over to the lower dimensional setting. At the same time, the (2 + 1) dimensional model is vastly simpler, and one can actually write down candidates for a quantum gravity [1].

Another important feature of the conformally invariant scalar theory in three dimensions is that its ϕ^6 coupling can in principle, induce a divergence in the four-point Green's functions neccessitating a ϕ^4 coupling, which is not conformally invariant. There is no analogue of this possibility in four dimensions, the renormalisable ϕ^4 coupling is conformally invariant and cannot generate any divergence that corresponds to a nonconformally invariant coupling [3].

Quantum fields [4] have profound influence on the dynamical behaviour of the early universe [5-9]. In the present work we discuss the finite temperature phase transitions in a (2+1) dimensional curved spacetime for ϕ^6 model. In Ref. [9] we have obtained a finite expression for the one-loop effective potential [10] using the ϕ^6 model in a (3+1) dimensional Bianchi type-I

spacetime.

Consider a massive self interacting scalar field ϕ coupled arbitrarily to the gravitational back ground and described by the Lagrangian density $\pounds$,

$$\pounds = \sqrt{-g}\left\{\frac{1}{2}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \xi R\phi^2] - \frac{1}{2}\lambda^2\phi^2(\phi^2 - m/\lambda)^2\right\} \tag{1}$$

The equation of motion associated with the Lagrangian(1) is,

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + (m^2 + \xi R)\phi - 4\kappa\phi^3 + 3\lambda^2\phi^5 = 0 \tag{2}$$

in which we put $m\lambda = \kappa$. We can write $\phi = \phi_c + \phi_q$ where ϕ_c is the classical field and ϕ_q is a quantum field with vanishing vacuum expectation value, $< \phi_q > = 0$. The field equation for the classical field ϕ_c is,

$$\begin{aligned} g^{\mu\nu}\nabla_\mu\nabla_\nu\phi_c + [(m_r^2 + \delta m^2) + (\xi_r + \delta\xi)R]\phi_c - 4(\kappa_r + \delta\kappa)\phi_c^3 - 12(\kappa_r + \delta\kappa)\phi_c < \phi_q^2 > \\ + 3(\lambda_r^2 + \delta\lambda^2)\phi_c^5 + 30(\lambda_r^2 + \delta\lambda^2)\phi_c^3 < \phi_q^2 > +15(\lambda_r^2 + \delta\lambda^2)\phi_c < \phi_q^4 > = 0 \end{aligned} \tag{3}$$

where the bare parameters m,ξ, κ and λ are replaced by the renormalised terms [9]. To the one loop quantum effect, the field equation for the quantum field ϕ_q is,

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi_q + (m_r^2 + \xi R)\phi_q - 12\kappa_r\phi_c^2\phi_q + 15\lambda_r^2\phi_c^4\phi_q = 0 \tag{4}$$

The effective potential V_{eff} is given by,

$$V_{eff} = \frac{1}{2}[(m_r^2 + \delta m^2) + (\xi_r + \delta\xi)R][\phi_c^2 + <\phi_q^2>] - (\kappa_r + \delta\kappa)\phi_c^4$$
$$-6(\kappa_r + \delta\kappa)\phi_c^2 <\phi_q^2> -(\kappa_r + \delta\kappa) <\phi_q^4> + \frac{1}{2}(\lambda_r^2 + \delta\lambda^2)\phi_c^6$$
$$+\frac{15}{2}(\lambda_r^2 + \delta\lambda^2)\phi_c^4 <\phi_q^2> + \frac{15}{2}(\lambda_r^2 + \delta\lambda^2)\phi_c^2 <\phi_q^4> + \frac{1}{2}(\lambda_r^2 + \delta\lambda^2) <\phi_q^6> \quad (5)$$

To make V_{eff} finite, the following renormalisation conditions are used,

$$m_r^2 = \left(\frac{\partial^2 V_{eff}}{\partial\phi_c^2}\right)_{\phi_c = R = 0}, \qquad \xi_r = \left(\frac{\partial^3 V_{eff}}{\partial R \partial\phi_c^2}\right)_{\phi_c = R = 0},$$
$$\kappa_r = \left(\frac{\partial^4 V_{eff}}{\partial\phi_c^4}\right)_{\phi_c \ = R = 0}, \qquad \lambda_r^2 = \left(\frac{\partial^6 V_{\mathrm{eff}}}{\partial\phi_c^6}\right)_{\phi_c = R = 0} \quad (6)$$

To evaluate $<\phi_q^2>$, $<\phi_q^4>$, and $<\phi_q^6>$ we adopt the canonical quantisation relations:

$$[\phi_q(t,x), \phi_q(t,y)] = [\pi_q(t,x), \pi_q(t,y)] = 0; \qquad [\phi_q(t,x), \pi_q(t,y)] = i\delta^3(x-y) \quad (7)$$

where the conjugate momentum π_q is defined by $\pi_q = \frac{\partial \pounds}{\partial(\partial_i\phi)}$.

We consider a (2+1) dimensional Bianchi type-I spacetime with small anisotropy which has the line element

$$ds^2 = C(\eta)d\eta^2 - a_1^2(\eta)dx^2 - a_2^2(\eta)dy^2, \qquad C = a_1 a_2 \quad (8)$$

In this model the mode function of the quantum field ϕ_q can be written in the separated form as $u_k = C^{-1/4}(2\pi)^{-1}\exp(i\kappa.x)\chi_k(\eta)$. The wave equation

Eq. (4) will then lead to

$$\ddot{\chi} + \left\{ C \left[m_r^2 + (\xi_r - \frac{1}{8})R - 12\kappa_r \phi_c^2 + 15\lambda_r^2 \phi_c^4 + \sum_i \frac{k_i^2}{a_i^2} \right] + Q \right\} \chi_k = 0 \quad (9)$$

where the spacetime curvature function R and the anisotropic function Q are

$$R = 8C^{-1}(\dot{H} + H^2 + Q), \qquad H = \sum_i h_i, \qquad h_i = \frac{\dot{a}_i}{a_i}, \qquad Q = \frac{1}{64} \sum_{i<j} (h_i - h_j)^2 \quad (10)$$

When the metric is slowly varying Eq. (9) possesses WKB approximation solution:

$$\chi_k = (2W_k)^{-\frac{1}{2}} \exp(-i \int d\eta W_k) \quad (11)$$

where, $$W_k = \left\{ C \left[m_r^2 + (\xi_r - \tfrac{1}{8})R - 12\kappa_r \phi_c^2 + 15\lambda_r^2 \phi_c^4 + \sum_i \tfrac{k_i^2}{a_i^2} \right] + Q \right\}^{\frac{1}{2}}$$

Using the above solution we get:

$$< \phi_q^2 >= \tfrac{1}{8\pi^2 C(\eta)} \int d^2k \left[m_r^2 + (\xi_r - \tfrac{1}{8})R - 12\kappa_r \phi_c^2 + 15\lambda_r^2 \phi_c^4 + \sum_i \tfrac{k_i^2}{a_i^2} + \tfrac{Q}{C} \right]^{-1/2}$$

$$= \tfrac{1}{16\pi} \left[\Lambda + \tfrac{(m_r^2 + (\xi_r - \frac{1}{8})R - 12\kappa_r \phi_c^2 + 15\lambda_r^2 \phi_c^4 + \frac{Q}{C})}{2\Lambda} - (m_r^2 + (\xi_r - \tfrac{1}{8})R - 12\kappa_r \phi_c^2 + 15\lambda_r^2 \phi_c^4 + \tfrac{Q}{C})^{1/2} \right] \quad (12)$$

and similarly,

$$< \phi_q^4 >= \tfrac{1}{128\pi^3 C} \log \left[1 + \tfrac{\Lambda^2}{(m_r^2 + (\xi_r - \frac{1}{8})R - 12\kappa_r \phi_c^2 + 15\lambda_r^2 \phi_c^4 + \frac{Q}{C})} \right] \quad (13)$$

where we have introduced a momentum cut-off Λ to regularise the k-integration. From the renormalisation conditions given by Eq. (6) the renormalisation

counter terms are evaluated and substituting these terms we find $\frac{\partial V_{eff}}{\partial \phi_c}$ obtained from Eq. (5) as,

$$\frac{\partial V_{eff}}{\partial \phi_c} = (m_r^2 + \xi_r R)\phi_c - \left[\frac{3\lambda_r^2[(m_r^2+\frac{Q}{C})-A]}{4B}\right]\phi$$

$$- \left[\frac{D(m_r^2+\frac{Q}{C})\log[\frac{(m_r^2+\frac{Q}{C})}{A}]}{2\pi CEB}\right]\phi_c + \left[\frac{-3\lambda_r^2}{8B} + \frac{D}{2\pi CEB}\right](\xi_r - \tfrac{1}{8})R\phi_c \tag{14}$$

$$-\frac{4(m_r^2+\frac{Q}{C})^{1/2}}{B}\left[\pi\lambda_r^2 + \frac{2D(m_r^2+\frac{Q}{C})^{1/2}A^{1/2}}{E}\right]\phi_c^3 + \frac{64D(m_r^2+\frac{Q}{C})}{5EB}\phi_c^5$$

where,

$$A = (m_r^2 + (\xi_r - \tfrac{1}{8})R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \tfrac{Q}{C}), \qquad B = \left[\frac{-1350\lambda_r^2}{4} + \frac{3420\kappa_r^2}{2(m_r^2+\frac{Q}{C})}\right],$$

$$D = \left[\lambda_r^2 - \frac{54\kappa_r\lambda_r^2}{4\pi(m_r^2+\frac{Q}{C})^{1/2}} + \frac{135\kappa_r^2}{2\pi(m_r^2+\frac{Q}{C})^{3/2}}\right] and \quad E = \left[-(m_r^2 + \tfrac{Q}{C})^{1/2} + \frac{3\kappa_r}{4\pi(m_r^2+\frac{Q}{C})}\right] \tag{15}$$

Thus it is clear that we can obtain a finite expression for the one loop effective potential for the ϕ^6 model in (2+1) dimensional Bianchi Type I spacetime. In a previous work [9] we have shown that ϕ^6 potential can be regularised in (3+1) dimensional curved spacetime. In the present work using the momentum cutoff technique we obtain a divergenceless expression for the ϕ^6 potential in a (2+1) dimensional Bianchi type-I background spacetime.

While constructing a theory of the interaction between quantised matter fields and a classical gravitational field one has to identify the energy-momentum tensor of the quantised fields which acts as the source of the

gravitational field. The energy-momentum tensor [4] for the present ϕ^6 field is

$$T_{\mu\nu} = (1-2\xi)\partial_\mu\phi\partial_\nu\phi + (2\xi - \tfrac{1}{2})g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi - 2\xi\phi\nabla_\mu\nabla_\nu\phi + 2\xi g_{\mu\nu}\phi\Box\phi - \xi G_{\mu\nu}\phi^2$$
$$+(\tfrac{m^2}{2})g_{\mu\nu}\phi^2 - 2\kappa g_{\mu\nu}\phi^4 + \tfrac{3}{2}\lambda^2 g_{\mu\nu}\phi^6 \qquad (16)$$

where $G_{\mu\nu}$ is the Einstein tensor. The expection value of the energy-momentum tensor can be broken into classical and quantum parts. The energy-momentum tensor for the classical part is obtained by substituting ϕ_c for ϕ in Eq. (16). The $\eta\eta$ component of the classical renormalized energy-momentum tensor is given by,

$$\langle T^\eta_\eta\rangle^C = \frac{1}{2C}\phi_c^{'}\phi_c^{'} + 2\xi\frac{C^{'}}{C^2}\phi_c\phi_c^{'} + \frac{3\xi}{C}(\frac{C^{'}}{C}+k)\phi_c^2 + \frac{m^2}{2}\phi_c^2 - 2\kappa\phi_c^4 + \frac{3}{2}\lambda^2\phi_c^4 \qquad (17)$$

where k=+1,0,-1 corresponds to the case of positive, zero or negative spatial curvature respectively. The quantum part of is $\langle T_{\mu\nu}\rangle$ is

$$\langle T_{\mu\nu}\rangle^Q = (1-2\xi)\langle\partial_\mu\phi_q\partial_\nu\phi_q\rangle + (2\xi-\tfrac{1}{2})g_{\mu\nu}\langle\partial_\alpha\phi_q\partial^\alpha\phi_q\rangle - 2\xi\langle\phi_q\nabla_\mu\nabla_\nu\phi_q\rangle$$
$$+2\xi g_{\mu\nu}\langle\phi_q\Box\phi_q\rangle - \xi G_{\mu\nu}\langle\phi_q^2\rangle + (\tfrac{m^2}{2})g_{\mu\nu}\langle\phi_q^2\rangle - 12\kappa g_{\mu\nu}\phi_c^2\langle\phi_q^2\rangle$$
$$+\tfrac{45}{2}\lambda^2 g_{\mu\nu}\phi_c^4\langle\phi_q^2\rangle + \tfrac{45}{2}\lambda^2 g_{\mu\nu}\phi_c^2\langle\phi_q^4\rangle \qquad (18)$$

To obtain the physically finite energy-momentum tensor of the system we can regularize the theory [11]. The finite expression for the expectation value of the quantum energy-momentum tensor is obtained as,

$$
\begin{aligned}
\langle T^{\eta}_{\eta}\rangle^{Q} = & \tfrac{C'^2[3A-\frac{Q}{C}]}{768\pi C^4 A^{3/2}} - \tfrac{1}{48\pi}A^{3/2} - \tfrac{C'^2 A^{1/2}}{256\pi C^3} - \tfrac{C'^2(A+\frac{Q}{C})}{128\pi C^3 A^{1/2}} \\
& + \tfrac{C'^2}{16\pi C^3}\left\{\xi_r - \tfrac{3(\xi_r-\frac{1}{8})\lambda_r^2}{8B} + \tfrac{D(\xi_r-\frac{1}{8})}{2CBE}\right\}\tfrac{(A+\frac{Q}{C})}{A^{1/2}} \\
& + \tfrac{C'}{16\pi C^2}[\tfrac{C'}{C} - 3]\left\{\xi_r - \tfrac{3(\xi_r-\frac{1}{8})\lambda_r^2}{8B} + \tfrac{D(\xi_r-\frac{1}{8})}{2CBE}\right\}A^{1/2} \\
& + \tfrac{3(m_r^2+\frac{Q}{C})}{4B}\left\{\lambda_r^2 - \tfrac{D\log(m_r^2+\frac{Q}{C})}{\pi CE}\right\} - \tfrac{6D(m_r^2+\frac{Q}{C})^{3/2}}{BE}
\end{aligned}
\tag{19}
$$

where A, B and E are defined by Eq. (15). It is clear that the energy-momentum tensor depends on the anisotropy of the spacetime.

To evaluate the finite temperature effective potential, the vaccum expectation value is replaced by the thermal average $< \phi >_T = \sigma_T$ taken with respect to a Gibbs ensemble [12]. Considering the same Lagrangian density as above, the zero loop effective potential is temperature independent,

$$V_0(\sigma) = \frac{1}{2}\xi R\sigma^2 + \frac{1}{2}\lambda^2\sigma^2(\sigma^2 - m/\lambda)^2 \tag{20}$$

The one loop approximation to finite temperature effective potential [10,12-14] is given by,

$$
\begin{aligned}
V_1^{\beta}(\sigma) &= \tfrac{1}{2\beta}\textstyle\sum_n \int \tfrac{d^2k}{(2\pi)^2}\ln(k^2 - M^2) \\
&= \tfrac{1}{2\beta}\textstyle\sum_n \int \tfrac{d^2k}{(2\pi)^2}\ln(\tfrac{-4\pi^2 n^2}{\beta^2} - E_M^2)
\end{aligned}
\tag{21}
$$

$$where, E_M^2 = k^2 + M^2, \qquad M^2 = m^2 + \xi R - 12\lambda m\sigma^2 + 15\lambda^2\sigma^4 \tag{22}$$

In the high temperature limit [13] we find that

$$V_1^{\beta}(\sigma) = \frac{1}{4\pi\beta^3}\xi(z) - \frac{M^2}{8\pi\beta}\ln M \tag{23}$$

where $\xi(z)$ is the Riemannian Zeta function. In the (2+1) dimensional case also, the symmetry breaking present in this ϕ^6 model can be removed if the temperature is raised above a certain value called the critical temperature. The order parameter of the theory is temperature dependent. The temperature dependence of finite temperature effective potential leads to phase transitions in the early universe.

On shifting the field from ϕ to $\phi{+}\sigma$ in the Eq. (2) and taking the Gibbs average of the corresponding equation we get:

$$\Box\sigma_T + (m^2 + \xi R)\sigma_T - 4\kappa\sigma_T{}^3 - 12\kappa\sigma_T < \phi^2 > -12\kappa\sigma_T^2 < \phi > +15\lambda^2\sigma_T < \phi^4 >$$
$$+30\lambda^2\sigma_T^2 < \phi^3 > +30\lambda^2\sigma_T^3 < \phi^2 > +15\lambda^2\sigma_T^4 < \phi > +3\lambda^2\sigma_T^5 = 0 \tag{24}$$

Using the standard finite temperature Green's- function methods we can find that in the high temperature limit, $< \phi^2 >= \frac{-m}{4\pi T}$ and from similar calculations, $< \phi^4 >= \frac{T}{4\pi m}$, $< \phi^3 >= 0$ and $< \phi >= 0$. Thus Eq. (24) becomes

$$\Box\sigma_T + (m^2 + \xi R)\sigma_T - 4\kappa\sigma_T^3 + \frac{3\kappa\sigma_T m}{\pi T} + \frac{15\lambda^2\sigma_T T}{4\pi m} - \frac{15\lambda^2\sigma_T^3 m}{2\pi T} + 3\lambda^2\sigma_T^5 = 0 \tag{25}$$

Assuming that σ_T is a constant we obtain,

$$\sigma_T \left[(m^2 + \xi R) - 4\kappa\sigma_T^2 + \frac{3\kappa m}{\pi T} + \frac{15\lambda^2 T}{4\pi m} - \frac{15\lambda^2\sigma_T^2 m}{2\pi T} + 3\lambda^2\sigma_T^4 \right] = 0 \quad (26)$$

This equation has degenerate solutions:

$$\sigma_T = 0, and \quad \sigma_T^2 = \frac{\left\{ (4\kappa + \frac{15\lambda^2 m}{2\pi T}) \pm \left(4\lambda^2 m^2 - 12\lambda^2\xi R + \frac{24\lambda^3 m^2}{\pi T} + \frac{225\lambda^4 m^2}{4\pi^2 T^2} - \frac{45\lambda^4 T}{\pi m} \right)^{\frac{1}{2}} \right\}}{6\lambda^2} \quad (27)$$

Each solution of these equations defines a possible phase of the field system with its characteristic excitations. On heating the field system from absolute zero, the two branches of σ_T^2 given by the above equation coincide at a temperature for which

$$\left(4\lambda^2 m^2 - 12\lambda^2\xi R + \frac{24\lambda^3 m^2}{\pi T} + \frac{225\lambda^4 m^2}{4\pi^2 T^2} - \frac{45\lambda^4 T}{\pi m} \right)^{\frac{1}{2}} = 0 \quad (28)$$

yielding a common value of σ_T. The existence of the separate branches of σ_T^2 implies that the phase transition is of first order [12,15]. From Eq. (27) it is clear that the order parameter does not vanish even for very high values of the temperature. Fig. 1 gives the variation of the two branches of σ_T^2 with respect to temperature. It is found that the two branches coincides at a particular value of T given by Eq. (28). From the graphs it is clear that there is a discontinuity for the variation of the order parameter with temperature, indicating a first order phase transition. For a second order

phase transition the order parameter will be a continuous function of T and it decreases smoothly with increasing temperature and vanishes at T_c [12].

The characteristic of a first order phase transition is the existence of a barrier between the symmetric and the broken phase [16]. The temperature dependence of V_{eff} for a first order phase transition obtained using the present ϕ^6 model in the (2+1) dimensional background spacetime is shown in Fig. 2. It is found that for $T \gg T_c$, the effective potential attains a minimum at $\sigma = 0$, which corresponds to the completely symmetric case. When the temperature decreases, a global minimum appears at $\sigma = 0$ and two local minima at $\sigma \neq 0$, which shows the existence of a barrier between the global and local minima (Fig. 2a). At $T=T_c$, all the minima are degenerate, which implies that the symmetry is broken (Fig. 2b). For T¡T_c the minima at $\sigma \neq 0$ become global minima (Fig. 2c). If for T$\leq T_c$ the extremum at $\sigma = 0$ remains a local minimum, there must be a barrier between the minimum at $\sigma = 0$ and at $\sigma \neq 0$. Therefore the change in σ in going from one phase to the other must be discontinuous, indicating a first order phase transition [12,16].

Fig. 3 clearly shows the crucial role of scalar curvature R in determining the fate of symmetry and the phase transions for the present model. From the figure it is clear that the first order phase transition takes place as R

changes. It is found that for $R = 0$ or $\xi = 0$ the system remains in the symmetry broken state for all values of T$\leq T_c$. As the temperature is increased above T_c, the symmetry is restored depending on the values of R and ξ also. It is also found that symmetry can be restored either by increasing the value of R or by increasing the value of ξ keeping the temperature constant, even below the critical temperature. It is clear from Fig. 4 that there is a barrier between the symmetric and broken phases. Thus phase transition, induced by the coupling constant ξ is also of first order. This shows that the scalar-gravitational coupling and the scalar curvature do play a crucial role in determining the nature of phase transitions took place in the early universe.

A first order phase transition proceeds by nucleation of bubbles of broken phase in the background of unbroken phase [12]. Bubble nucleation can have various consequences in the early universe. The bubbles expand and eventually collide, while new bubbles are continuously formed, until the phase transition is completed.

While discussing the bubble collisions [17-18], one has to consider the interaction between the bubble field and the surrounding plasma. Considering the form of Lagrangian as in Eq. (1) for a massive self interacting com-

plex field Φ coupled arbitrarily to the gravitational back ground, the from of potential is

$$V(\phi) = \frac{1}{2}\xi R \left|\Phi\right|^2 + \frac{1}{2}\lambda^2 \left|\Phi^2\right| (\left|\Phi\right|^2 - m/\lambda)^2 \tag{29}$$

with a minimum at $|\Phi| = 0$ and a set of minima at $|\Phi| = \left[\frac{m\pm\sqrt{-\xi R}}{\lambda}\right]^{1/2}$, connected by U(1) transformation and towards which the false vacuum will decay via bubble nucleation. The equation of motion for this system is

$$\partial_\mu \partial^\mu \Phi = -\frac{\partial V}{\partial \Phi} \tag{30}$$

Let us consider the minimally coupled case $\xi = 0$. For the thin wall regime [12] the approximate solution of Eq. (30) is obtained as

$$|\Phi| = \left\{\frac{m}{2\lambda}\left[\tanh\left(m(\chi - R_0) + 1\right)\right]\right\}^{1/2} \tag{31}$$

where R_0 is the bubble radius at nucleation time and $\chi^2 =\mid \vec{x} \mid^2 - t^2$. Cosidering the damping effect of the surrounding plasma on the motion of the walls we insert a frictional term in the equation of motion ,

$$\partial_\mu \partial^\mu \Phi + \gamma \left|\dot{\Phi}\right| e^{i\theta} = -\frac{\partial V}{\partial \Phi} \tag{32}$$

where $\left|\dot{\Phi}\right| = \frac{\partial|\Phi|}{\partial t}$, θ is the phase of the field and γ stands for the friction coefficient.

To find the solution of Eq. (32) in the thin wall limit, first we suppose that solution for which the wall has the form of a travelling wave do exist.

Writing Φ in polar form $\Phi = \rho e^{i\theta}$, we can rewrite Eq. (32) and for a single bubble configuration we take the phase of the bubble θ to be constant. Then the equation for the modulus of the field is

$$\partial_\mu \partial^\mu \rho + \gamma \dot{\rho} = -\frac{\partial V(\rho)}{\partial \rho} \tag{33}$$

Following Ferrera and Melfo [18] we get

$$(1 - v_{ter}^2)\frac{\partial^2 \rho}{\partial r^2} = \frac{\partial V(\rho)}{\partial \rho} \tag{34}$$

where r is the radial coordinate and v_{ter} is the terminal velocity of the bubble walls. For the present ϕ^6 potential the solution for Eq. (34) is obtained as

$$\rho = \left\{ \frac{m}{2\lambda} \left[\tanh \left(\frac{m(r - v_{ter}t - R_0)}{\sqrt{1 - v_{ter}^2}} \right) + 1 \right] \right\}^{1/2} \tag{35}$$

which is simply a Lorentz-contracted moving domain wall. Thus it is clear that there exists an exact solution for the damped motion of the bubble in the thin wall regime.

Whether or not the Universe recovers from a first order phase transition and any relics are left behind depends upon the kinematics of bubble nucleation and on the process of eventual transition to the new phase.

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Figure Captions

1. Fig. 1 : Variation of the two branches of σ_T^2 with respect to temperature. The two curves coincides after the temperature which satisfies Eq. (48), where $m = 0.9371$, $\lambda = 0.008$, $R = 0.9$ and $\xi = 0.2$ in the figure.

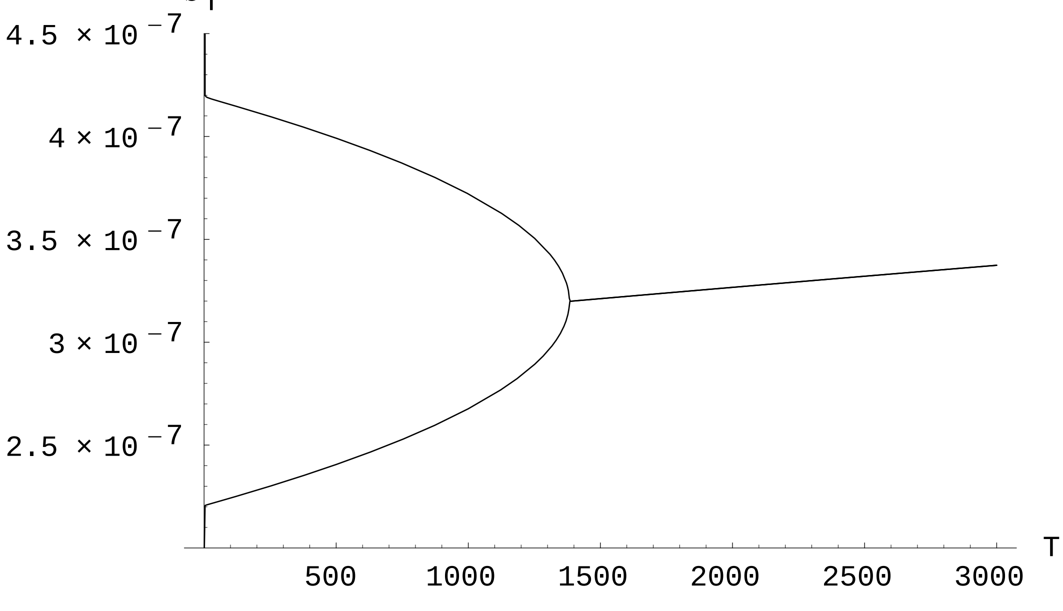
2. Fig. 2a : The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.008$, $R = 1.93, \xi = 0.098$ and $T = 16.5$ such that $\mathbf{T} > \mathbf{T}_c$.

3. Fig. 2b : The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.008$, $R = 0.42, \xi = 0.02$ and $T = 9$ such that $\mathbf{T} = \mathbf{T}_c$.

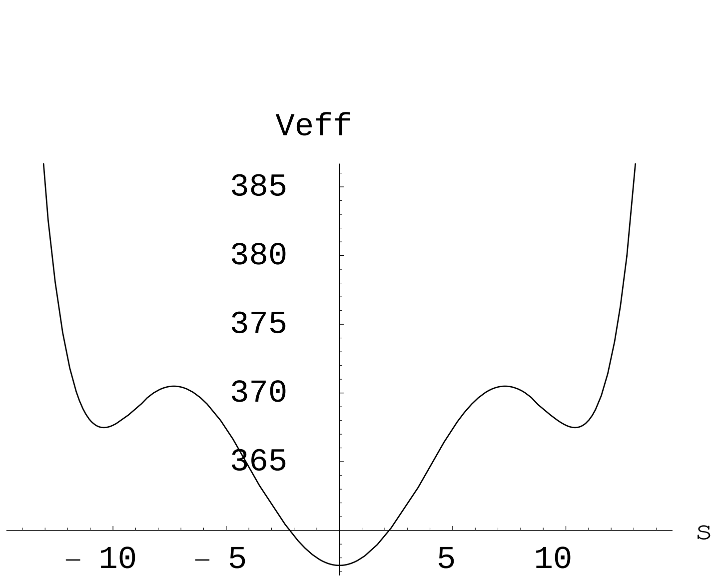
4. Fig. 2c : The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.008$, $R = 0.35, \xi = -0.3$ and $T = 5$ such that $\mathbf{T} < \mathbf{T}_c$.

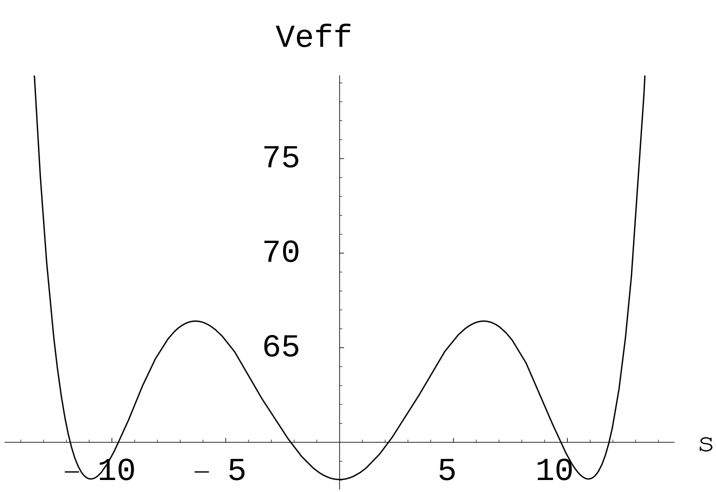
5. Fig. 3 : The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.009$, $\xi = 0.1$ and T= 4. Starting from top the curves corresponds to the following values of the curvature : R=20,3,0.99,0.02,-0.72.

6. Fig. 4 : The behaviour of finite temperature effective potential as a function of σ for fixed $m = 0.9371$, $\lambda = 0.009$, $R = 0.2$ and $T = 5$. Starting from top the curves corresponds to the following values of the curvature : ξ = 9,2,0.85,0.025,-0.35,-0.8.

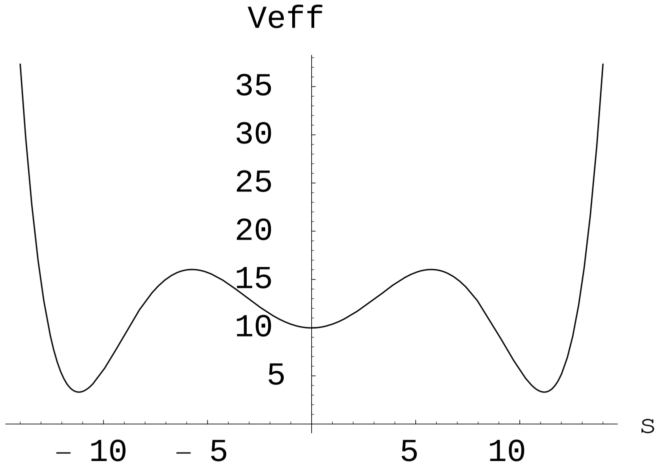


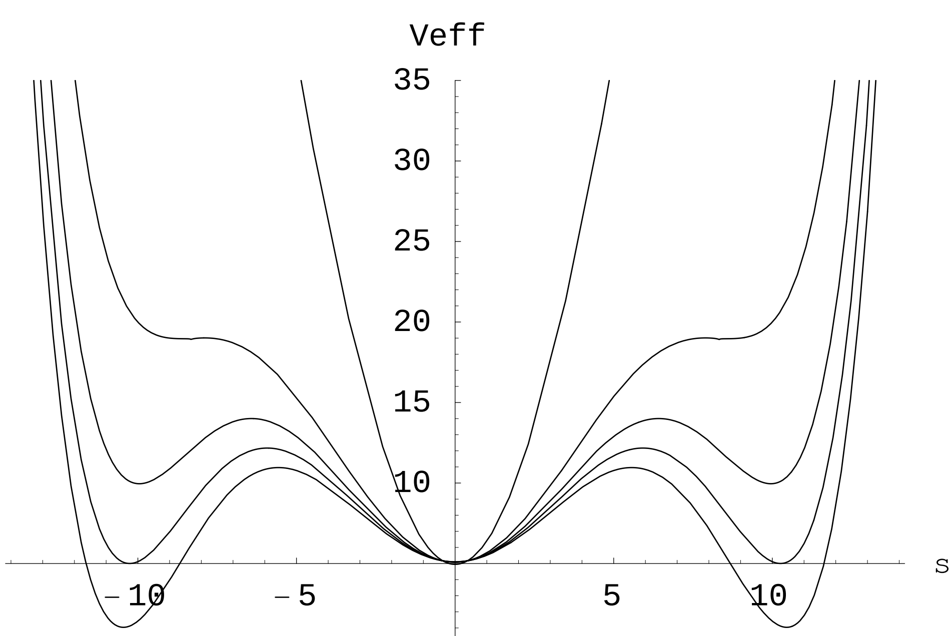
s_T^2
4.5×10^{-7}
4×10^{-7}
3.5×10^{-7}
3×10^{-7}
2.5×10^{-7}
500
1000
1500
2000
2500
3000
T


Veff
385
380
375
370
365
s
– 10
– 5
5
10

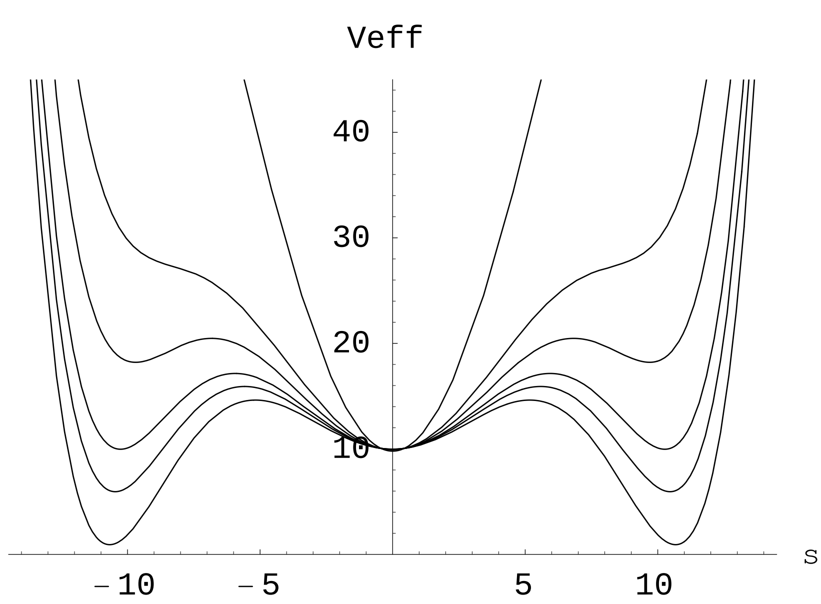


Veff
75
70
65
S
-10
-5
5
10


Veff
35
30
25
20
15
10
5
s
– 10
– 5
5
10



Veff
35
30
25
20
15
10
s
−10
−5
5
10



Veff
40
30
20
10
s
-10
-5
5
10